NBODY6 Overview

Hermite Integration

Neighbour Scheme

Regularization

Hierarchies

Stellar Evolution

Post-Newtonian

Data Structure

Collab: S. Mikkola, K. Nitadori, J. Hurley

Hermite Integration

Taylor series for $\mathbf{F}$ and $\mathbf{F}^{(1)}$

$$\begin{aligned}
\mathbf{F} &= \mathbf{F}_0 + \mathbf{F}_0^{(1)}\, t + \frac{1}{2}\mathbf{F}_0^{(2)}\, t^2 + \frac{1}{6}\mathbf{F}_0^{(3)}\, t^3 \\
\mathbf{F}^{(1)} &= \mathbf{F}_0^{(1)} + \mathbf{F}_0^{(2)}\, t + \frac{1}{2}\mathbf{F}_0^{(3)}\, t^2
\end{aligned}$$

Prediction

$$\begin{aligned}
\mathbf{r}_j &= ((\frac{1}{6}\mathbf{F}_0^{(1)}\, \delta t'_j + \frac{1}{2}\mathbf{F}_0)\, \delta t'_j + \mathbf{v}_0)\, \delta t'_j + \mathbf{r}_0 \\
\mathbf{v}_j &= ((\frac{1}{2}\mathbf{F}_{\mathbf{0}}^{(1)}\, \delta t'_j + \mathbf{F}_0)\, \delta t'_j + \mathbf{v}_0\, ; \qquad \delta t'_j = t - t_0
\end{aligned}$$

New forces $\qquad \mathbf{F},\ \mathbf{F}^{(1)}$

Higher derivatives

$$\begin{aligned}
\mathbf{F}_0^{(3)} &= (2(\mathbf{F}_0 - \mathbf{F}) + (\mathbf{F}_0^{(1)} + \mathbf{F}^{(1)})\, t)\, \frac{6}{t^3} \\
\mathbf{F}_0^{(2)} &= (-3(\mathbf{F}_0 - \mathbf{F}) - (2\mathbf{F}_0^{(1)} + \mathbf{F}^{(1)})\, t)\, \frac{2}{t^2}
\end{aligned}$$

Corrector for i

$$\begin{aligned}
\Delta \mathbf{r}_i &= \frac{1}{24}\mathbf{F}_0^{(2)} \Delta t^4 + \frac{1}{120}\mathbf{F}_0^{(3)}\, \Delta t^5 \\
\Delta \mathbf{v}_i &= \frac{1}{6}\mathbf{F}_0^{(2)} \Delta t^3 + \frac{1}{24}\mathbf{F}_0^{(3)}\, \Delta t^4
\end{aligned}$$

Basic Regularization

Two-body equation $\ddot{x} = -\frac{M}{x^2}$

Smoothing function $t' \equiv \frac{dt}{d\tau} = x$

Rule of differentiation $\frac{d}{dt} = \frac{1}{x}\frac{d}{d\tau}$

Time-smoothed equation $x'' = \frac{x'^2}{x} - M$

Binding energy $h = \frac{1}{2}\dot{x}^2 - \frac{M}{x}$

Substitution $\dot{x} = \frac{x'}{x} \Rightarrow$ $x'' = 2hx + M$

Coordinate transformation $u^2 = x$

Twice diff. of u^2 and $h \Rightarrow$ $u'' = \frac{1}{2}hu$

Regular equation for $x \Rightarrow 0$

KS Regularization

New coordinates $R = u_1^2 + u_2^2 + u_3^2 + u_4^2$

Time transformation $dt = R\, d\tau$

Coordinate transformation $\mathbf{R} = \mathcal{L}(\mathbf{u})\, \mathbf{u}$

Levi-Civita matrix

$$\mathcal{L}(\mathbf{u}) = \begin{bmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \end{bmatrix}$$

Equations of motion

$$\begin{aligned} \mathbf{u}'' &= \frac{1}{2} h\, \mathbf{u} + \frac{1}{2} R\, \mathcal{L}^{\mathcal{T}}\, \mathbf{P} \\ h' &= 2\, \mathbf{u}' \cdot \mathcal{L}^{\mathcal{T}}\, \mathbf{P} \\ t' &= \mathbf{u} \cdot \mathbf{u} \end{aligned}$$

Close encounter $\Delta t_i < \Delta t_{cl}\,; \quad R < r_{cl}$

Termination $\gamma \equiv \dfrac{|\mathbf{P}|\, R^2}{m_i + m_j} > 0.5$

Centre of mass motion $\ddot{\mathbf{r}} = \dfrac{m_i\, \mathbf{P}_i + m_j\, \mathbf{P}_j}{m_i + m_j}$

Perturber selection $r_k < \lambda\, R\,, \quad \gamma > 1 \times 10^{-6}$

Stellar Evolution

Stellar HR types	$K^* = 0, ..., 15$
Fast look-up (Pop I & II)	$r^*(t),\ m_c(t),\ L^*(t),\ K^*(t)$
Wind mass loss	$\dot{m} = -2 \times 10^{-13} r^* L^*/m$
Single stars	$\Delta m/m > 1\%$, new r^*
Updating times	$T_{\rm ev} = t + \min(\Delta t_{\rm ev}, \Delta t_{\rm rem})$
Stellar rotation	$\Delta J_{\rm spin} = 2\Delta m r^2 \Omega_{\rm rot}/3$
White dwarfs	cooling curves, $\Delta t_{\rm ev} = 10^6\,{\rm yr}$
Supernova outburst	$m_{\rm c} > m_{\rm chandra} \Rightarrow$ SN
NS velocity kick	$v >> v_\infty \sim 2\,km/s$
Binary mass loss	$m\,a = {\rm const}$
Synthetic HR diagram	binaries and single stars
Energy conservation	$\Delta E = \Delta m\,(\frac{1}{2} v^2 + \Phi)$

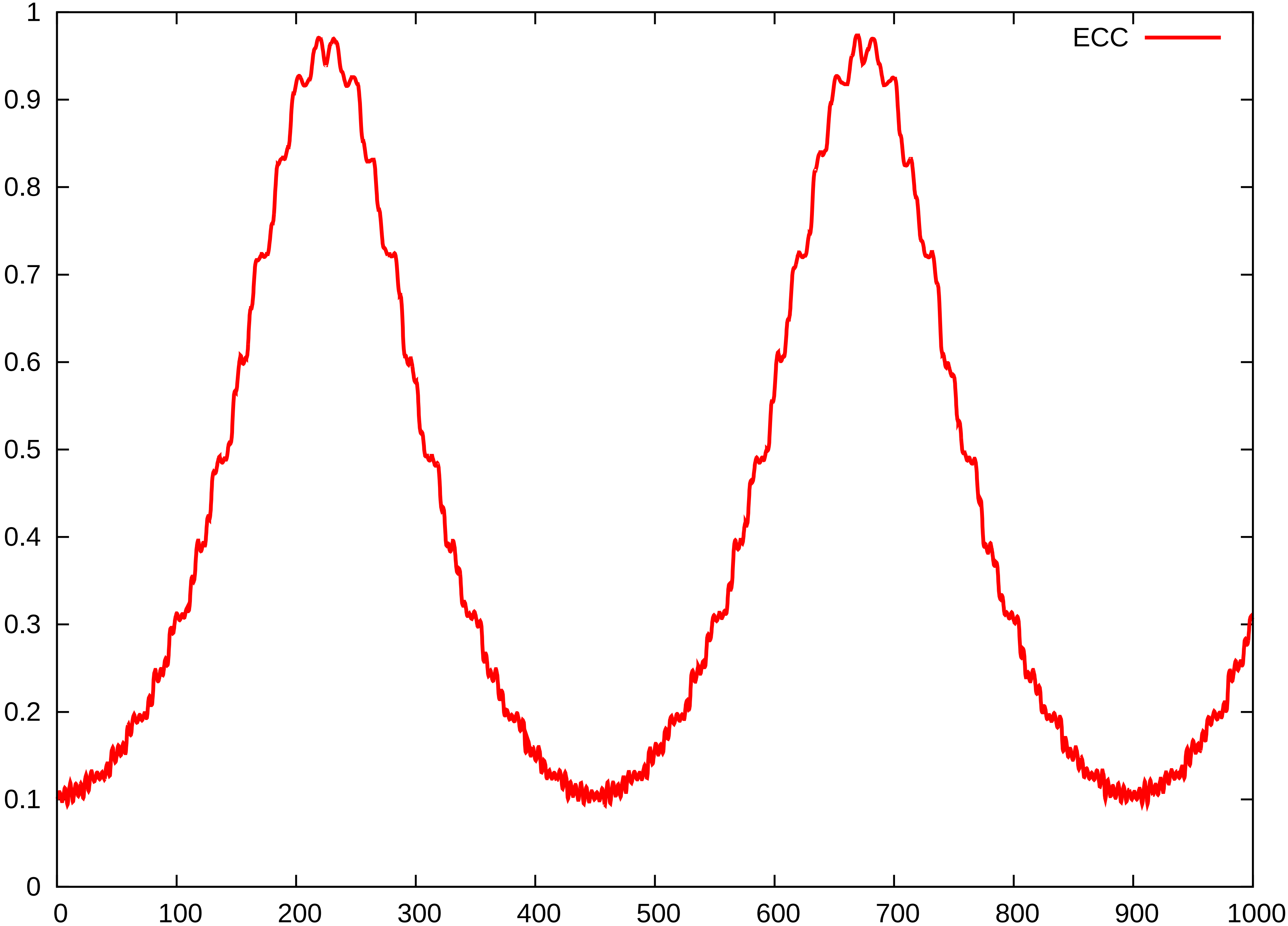
Three-Body Dynamics

Basic interactions	$B + S \Rightarrow \tilde{B} + \tilde{S}$
Fast escapers	Resonance or radial intruder
Formation of hierarchy	$B + B \Rightarrow \tilde{B} + S + \tilde{S} \Rightarrow T + \tilde{S}$
Hierarchical stability	Constant inner period
Induced collision	Kozai cycles $\Rightarrow e_{\rm max}$
Gravitational slingshot	Internal/external effects

PN Scenarios

Unperturbed binaries	BH or NS
Globular clusters	BH or NS + N^*
Galactic centres	IMBH + BH + N^*
Supermassive systems	SMBH + IMBH + N^*

Energy considerations	$\dfrac{m_1 m_2}{2a^*} = \kappa \lvert E_{\rm tot} \rvert, \quad E_{\rm tot} = -0.25$
Super-hard binary	$m_1 = 10\bar{m}, \quad \kappa = 0.1, \quad a^* = \dfrac{2000}{N^2}$
Schwarzschild radius	$R_{\rm Sch} = \dfrac{2M}{c^2}$
GR radiation time-scale	$t_{\rm GR} \propto \dfrac{c^5 a^5}{m_1^2}(1-e^2)^{7/2}, \quad c = \dfrac{3 \times 10^5}{V^*}$



ECC
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0.1
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1
0
100
200
300
400
500
600
700
800
900
1000

Data Structure

Single stars	$2\,N_{\rm p} < i \le N, \quad \mathcal{N}_i = i$
KS pairs	$1 \le i \le 2\,N_{\rm p}\,, \quad i_p = i_{\rm icm} - N$
C.m. particles	$i > N, \quad \mathcal{N} = N_0 + \mathcal{N}_k\,, \quad k = 2i_p - 1$
Stable triples	KS + ghost, $\mathcal{N}_{\rm cm} = -\mathcal{N}_k$
Ghost particles	$\mathcal{N}_{\rm ghost} = \mathcal{N}_{2i_{\rm p}-1}\,, \quad m_{\rm ghost} = 0$
Stable quadruples	KS + KS ghost, $\mathcal{N}_{\rm cm} = -\mathcal{N}_k$
Higher orders	T + KS, $\mathcal{N}_{\rm cm} = -(2N_0 + \mathcal{N}_k)$
Chain members	$2\,N_{\rm p} < i_{\rm cm} \le N, \quad \mathcal{N}_{\rm cm} = 0$
Single escape	$2N_{\rm p} < i \le N, \quad r_i > 2r_{\rm tide}\,,$ remove i
Binary escape	$i > N\,, \quad r_i > 2r_{\rm tide}\,, \quad 2i_p - 1,\ 2i_p$
Hierarchy escape	$i > N\,, \quad r_i > 2r_{\rm tide}\,, \quad 2i_p - 1,\ 2i_p\,,\ i_{\rm ghost}$

Essential Input Parameters

Particle numbers	$N,\ n_{\rm max},\ N_{\rm crit}$
Integration variables	$\eta_{\rm I},\ \eta_{\rm R},\ S_0,\ \Delta T,\ T_{\rm crit},\ Q_{\rm E},\ R_{\rm pc},\ \bar{m}$
Optional procedures	consult list of 50 choices, in define.f
KS parameters	$\Delta t_{\rm cl},\ R_{\rm cl},\ \eta_{\rm U},\ \gamma_{\rm min}$
IMF	$\alpha,\ m_1,\ m_N,\ N_{\rm b},\ \#20$
Virial theorem	$Q_{\rm V} = 0.5$ for equilibrium
Primordial binaries	$a_{\rm max},\ e_0,\ m_1/m_2,\ a_{\rm min},\ \#20$

Numerical examples

$N = 1000,\ n_{\rm max} = 95,\ \eta_{\rm I} = 0.02,\ \eta_{\rm R} = 0.02,$

$S_0 = 0.3,\ \Delta T = 2,\ T_{\rm crit} = 100,$

$Q_{\rm E} = 2 \times 10^{-5},\ R_{\rm pc} = 1,\ \bar{m} = 0.5$

$\#$ 1, 2, 5, 7, 14, 16, 20, 23

$\Delta t_{\rm cl} = 10^{-4},\ R_{\rm cl} = 0.001,\ \eta_{\rm U} = 0.2,\ \gamma_{\rm min} = 10^{-6}$

$\alpha = 2.3,\ m_1 = 10.0,\ m_N = 0.2,\ \#20 = 1$