

Initial Conditions

Coordinates: Plummer density or King model

Velocities: Isotropic or $f(E, \mathbf{J})$

Masses: Salpeter or Kroupa IMF

Primordial binaries: Observational evidence

Clumped subsystems: Hierarchical star formation

Initial segregation: Heavy stars near the centre

Gas expulsion: Imposed mass loss or SN

Intermediate MBH: GC or Galactic centre

Initial Scaling

Main input N, N_b, M_S, R_V

Initial data $m_i, \tilde{\mathbf{r}}_i, \tilde{\mathbf{v}}_i, \dots, i = 1, N$

Total energy $E = T - U$

Virial theorem $\mathbf{v}_i = q\tilde{\mathbf{v}}_i, q = \left[\frac{Q_V U}{T} \right]^{1/2}$

Units $G = 1, \sum m_i = 1, E_0 = -0.25$

Scaling $\hat{\mathbf{r}}_i = \frac{\mathbf{r}_i}{S^{1/2}}, \hat{\mathbf{v}}_i = \mathbf{v}_i S^{1/2},$
 $S = \frac{E_0}{q^2 T - U}$

Units

(a) Scaling relations

Length scale R_V in pc and M_S in M_\odot

Fiducial velocity $V^2 = GNM_S/R_V$

$$\tilde{V}^* = 1 \times 10^{-5} (GM_\odot/L^*)^{1/2} \text{ km/s}$$

$$L^* = 3 \times 10^{18} \text{ cm}$$

$$\text{Velocity unit } V^* = 6.5 \times 10^{-2} \left(\frac{NM_S}{R_V} \right)^{1/2} \text{ km/s}$$

$$\text{Fiducial time } \tilde{T}^* = (L^{*3}/GM_\odot)^{1/2} = 14.9 \text{ Myr}$$

$$\text{Time unit } T^* = \tilde{T}^* \left(\frac{R_V^3}{NM_S} \right)^{1/2} \text{ Myr}$$

(b) Conversion from N-body units

$$\tilde{r} = R_V r \text{ pc}, \quad \tilde{v} = V^* v \text{ km/s}$$

$$\tilde{t} = T^* t \text{ Myr}, \quad \tilde{m} = M_S m / \langle m \rangle M_\odot$$

$$\text{Crossing time } T_{\text{cr}} = 2\sqrt{2} T^* \text{ Myr}$$

Data Structure

Singles $2N_p < i \leq N, \mathcal{N}_i = i$

KS $1 \leq i \leq 2N_p, i_p = i_{\text{icm}} - N$

C.m. $i > N, \mathcal{N} = N_0 + \mathcal{N}_k$

Triple KS + ghost, $\mathcal{N}_{\text{cm}} = -\mathcal{N}_k$

Ghost $\mathcal{N}_g = \mathcal{N}_{2i_p}, m_g = 0$

Quad KS + KS ghost, $\mathcal{N}_{\text{cm}} = -\mathcal{N}_k$

Quint T + KS, $\mathcal{N}_{\text{cm}} = -(2N_0 + \mathcal{N}_k)$

Chain $2N_p < i_{\text{cm}} \leq N, \mathcal{N}_{\text{cm}} = 0$

Escape $2N_p < i \leq N, r_i > 2r_{\text{tide}}$

Binary $i > N, r_i > 2r_{\text{tide}}, 2i_p - 1, 2i_p$

Hierarchy $i > N, r_i > 2r_{\text{tide}},$
.
 $2i_p - 1, 2i_p, i_{\text{ghost}}$

Stellar Evolution

Stellar HR types $K^* = 0, \dots, 15$

Fast look-up $r^*(t), m_c(t), L^*(t), K^*(t)$

Wind mass loss $\dot{m} = -2 \times 10^{-13} r^* L^* / m$

Single stars Small $\Delta m / m$, new r^*

Updating times $T_{\text{ev}} = t + \min(\Delta t_{\text{ev}}, \Delta t_{\text{rem}})$

Stellar rotation $\Delta J_{\text{spin}} = \frac{2}{3} \Delta m r^2 \Omega_{\text{rot}}$

White dwarfs Cooling curves

Supernovae $m_c > 1.44 \Rightarrow \text{SN}, v \gg v_\infty$

Binary mass loss $ma = \text{const}$

Spin-orbit coupling $J_{\text{tot}} = J_{\text{orb}} + J_{\text{spin}}$

Tides Circularization and braking

Roche-lobe overflow $r^* > \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})} a$

Physical Collisions

Simple definition	$R_{\text{coll}} = \frac{3}{4}(r_1^* + r_2^*)$
Two-body encounter	KS regularization
Pericentre condition	$R'_0 R' < 0, \quad R < a$
Pericentre determination	Δt_{peri} from Kepler's equation
Predict \mathbf{R}_{peri} or iterate	$d\tau_0 = \frac{\Delta t_{\text{peri}}}{R}$, Newton-Raphson
Implement collision	$m_{\text{cm}} = m_1 + m_2, \quad r_{\text{cm}} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}$
Mass loss	$\Delta m = f(K_1^*, K_2^*)$
Initialize single body	$\mathbf{F}_1, \dot{\mathbf{F}}_1, \Delta t_1$
Compact subsystem	$\dot{R} \simeq 0$ by iteration
Transformation	$\mathbf{Q}, \mathbf{P} \Rightarrow \mathbf{r}, \dot{\mathbf{r}}$
New chain construction	$N_{\text{ch}} \Rightarrow N_{\text{ch}} - 1, \quad E_{\text{coll}} = E_{\text{ch}} - \mathcal{V}$

Primordial Binaries & Triples

Cluster parameters $r_h = 1.5$ pc, Salpeter IMF (0.2 – 10) M_\odot

Binary distribution $20 < a_b < 200$ and $8 < a_t < 80$ AU

Initial populations $N_s = 3500$, $N_b = 500$, $N_t = 100$

Mass factor $\frac{m_1 m_2}{2a_f} \simeq -E_{\text{tot}}$, $\Rightarrow a_f \simeq 10$ AU

Disruption $B + B \Rightarrow B' + S + S$

Exchange $B + S \Rightarrow \tilde{B} + \tilde{S}$

Stellar evolution $\dot{m} = f(m_0, Z, t)$, $\Rightarrow m_b a = \text{const}$

Collisions $a(1 - e) < 1.7 (m_b/2m_1)^{1/3} r_1^*$

Escape by recoil $B + S \Rightarrow \tilde{B} + \tilde{S}$, $v_\infty > \bar{v}$

Slingshot condition $v_\infty \simeq \left(\frac{G(m_1 + m_2)}{2a} \right)^{1/2}$

Program Control

Scheduling	Sorted list with $N^{1/2}$ members
Current time	$t_{\text{new}} = t_i + \Delta t_i, \quad i = \text{NEXT}(1)$
Next time	$T_{\text{min}} = \min(t_{\text{new}} + \Delta t_j)$
Next block	All $t_j + \Delta t_j = T_{\text{min}}$
Prediction	Full N or joint neighbour list
Irregular force	$\mathbf{F}_i, \mathbf{F}_i^{(1)}$ & corrector for n members
Regular force	$t_{\text{reg}} + \Delta t_{\text{reg}} = t_{\text{new}}, \quad \mathbf{F}_d, \mathbf{F}_d^{(1)}$
Continuity	Identical $\mathbf{F}_n, \mathbf{F}_n^{(1)}$ if no change
Strategy	Predicted coordinates for \mathbf{F}_{reg}

N-Body Scheduling

1. (Re-)Initialize times $\Delta t_{\min} \ \& \ t_{\min}, \quad i = 1, \dots, N$
2. Determine smallest level L_Q from $\Delta t_{\text{quant}}(L) = \Delta t_{\min}$
3. Enforce block-step search $t_L = t$
4. Count lowest levels $N(L), \quad [L_Q - 4, L_Q], \quad i = 1, N$
5. Sum levels backwards $\Sigma N(L) = N^{1/2}, \quad L = L^*$
6. Increase list interval $t_L + \Delta t(L^*) \Rightarrow t_L$
7. Form due soon list $t_i + \Delta t_i \leq t_L, \quad i = 1, \dots, N$
8. Record next time $t_{\min} = \min(t_i + \Delta t_i)$
9. Extract block members $t_i + \Delta t_i = t_{\min}, \quad i = 1, \dots, N_Q$
10. Set block time $t_{\text{block}} = t_k + \Delta t_k$
11. Check list renewal $t_{\text{block}} > t_L \Rightarrow \# 4$
12. Update next step $t_{\min} = \min(t_i + \Delta t_i), \quad i = 1, N_{\text{block}}$
13. Change of data structure $\Rightarrow \# 1$
14. Continue cycle $\Rightarrow \# 9$ or $\# 6$ (after new case)

Essential Input Parameters

Particle numbers	$N, n_{\max}, N_{\text{crit}}$
Integration variables	$\eta_{\text{I}}, \eta_{\text{R}}, S_0, \Delta T, T_{\text{crit}}, Q_{\text{E}}, R_{\text{pc}}, \bar{m}$
Optional procedures	consult list of 40 choices
KS parameters	$\Delta t_{\text{cl}}, R_{\text{cl}}, \eta_{\text{U}}, \gamma_{\text{min}}$
IMF	$\alpha, m_1, m_N, N_{\text{b}}, \#20$
Virial theorem	$Q_{\text{V}} = 0.5$ for equilibrium
Primordial binaries	$a_{\max}, e_0, m_1/m_2, a_{\min}, \#20$
Numerical examples	$N = 1000, n_{\max} = 70, \eta_{\text{I}} = 0.02, \eta_{\text{R}} = 0.03,$ $S_0 = 0.3, \Delta T = 2, T_{\text{crit}} = 100,$ $Q_{\text{E}} = 1 \times 10^{-5}, R_{\text{pc}} = 2, \bar{m} = 0.5$ $\# 1, 2, 5, 7, 14, 16, 20, 23$ $\Delta t_{\text{cl}} = 10^{-4}, R_{\text{cl}} = 0.001, \eta_{\text{U}} = 0.2, \gamma_{\text{min}} = 10^{-6}$ $\alpha = 2.3, m_1 = 10.0, m_N = 0.2, \#20 = 1$

NBODY6 Output

Control line T Q_V DE/E E_{tot} R_{cl} Δt_{min}

Main output T N NB KS NM MM NS $NSTEPS$ DE/E

Optional Procedures:

Cluster core N^2 algorithm for core radius and density centre

Lagrangian radii Percentile mass radii and half-mass radius

Error control Automatic error check and restart from last time

Escape Removal of distant members and table updates

Time offset Rescaling of all global times

Events Stellar types and energy partition

Binary analysis Regularized binary histograms and energy budget

Binary data bank Characteristic parameters for regularized binaries

HR diagram Evolutionary state of single stars and binaries

General data bank Detailed snapshots for data analysis

Integration Parameters

η_I	Time-step parameter for irregular force	0.02
η_R	Time-step parameter for regular force	0.03
S_0	Initial radius of the neighbour sphere	0.30
n_{\max}	Maximum neighbour number	70
Δt_{adj}	Time interval for energy check	2.0
Δt_{out}	Time interval for main output	10.0
Q_E	Tolerance for energy check	1×10^{-5}
R_V	Virial cluster radius (length unit) in pc	2.0
M_S	Mean stellar mass in solar units	0.5
Q_{vir}	Virial theorem ratio ($T/ U + 2W $)	0.5
Δt_{cl}	Time-step criterion for close encounters	1×10^{-4}
R_{cl}	Distance criterion for KS regularization	1×10^{-3}
η_U	Regularized time-step parameter	0.2
h_{hard}	Energy per unit mass for hard binary	1.0
γ_{\min}	Limit for unperturbed KS motion	1×10^{-6}
γ_{\max}	Termination criterion for soft binaries	0.001

Optional Procedures

- 1 Manual common save on unit 1 at any time
- 2 Common save on unit 2 at output time or restart
- 3 Data bank on unit 3 with specified frequency
- 5 Different types of initial conditions
- 7 Output of Lagrangian radii
- 8 Primordial binaries (extra input required)
- 10 Two-body regularization diagnostics
- 14 External tidal force; open or globular clusters
- 15 Multiple regularization or hierarchical systems
- 16 Updating of regularization parameters R_{cl} , Δt_{cl}
- 17 Modification of η_{I} and η_{R} by tolerance Q_{E}
- 19 Synthetic stellar evolution with mass loss
- 20 Different types of initial mass functions
- 23 Removal of distant escapers (isolated or tidal)
- 26 Slow-down of KS and/or chain regularization
- 27 Tidal circularization (sequential or continuous)
- 28 Magnetic braking and gravitational radiation
- 30 Chain regularization (with special diagnostics)

Basic Variables

\mathbf{x}_0	X0	Primary coordinates
\mathbf{v}_0	X0DOT	Primary velocity
\mathbf{x}	X	Prediction coordinates
\mathbf{v}	XDOT	Prediction velocity
\mathbf{F}	F	One half the total force (per unit mass)
$\mathbf{F}^{(1)}$	FDOT	One sixth the total force derivative
m	BODY	Particle mass (also initial mass m_0)
Δt	STEP	Irregular time-step
t_0	T0	Time of last irregular force
\mathbf{F}_I	FI	Irregular force
\mathbf{D}_I^1	FIDOT	First irregular force derivative
\mathbf{D}_I^2	D2	Second irregular force derivative
\mathbf{D}_I^3	D3	Third irregular force derivative
ΔT	STEPR	Regular time-step
T_0	T0R	Time of last regular forcex
\mathbf{F}_R	FR	Regular force
\mathbf{D}_R^1	FRDOT	First regular force derivative
\mathbf{D}_R^2	D2R	Second regular force derivative
\mathbf{D}_R^3	D3R	Third regular force derivative
R_s	RS	Neighbour sphere radius
L	LIST	Neighbour and perturber list

KS Variables

\mathbf{U}_0	U0	Primary regularized coordinates
\mathbf{U}	U	Regularized prediction coordinates
\mathbf{U}'	UDOT	Regularized velocity
\mathbf{F}_U	FU	One half the regularized force
\mathbf{F}'_U	FUDOT	One sixth the regularized force derivative
$\mathbf{F}_U^{(2)}$	FUDOT2	Second regularized force derivative
$\mathbf{F}_U^{(3)}$	FUDOT3	Third regularized force derivative
h	H	Binding energy per unit reduced mass
h'	HDOT	First derivative of specific binding energy
$h^{(2)}$	HDOT2	Second derivative of binding energy
$h^{(3)}$	HDOT3	Third derivative of binding energy
$h^{(4)}$	HDOT4	Fourth derivative of binding energy
$\Delta\tau$	DTAU	Regularized time-step
$t^{(2)}$	TDOT2	Second regularized derivative of time
$t^{(3)}$	TDOT3	Third regularized derivative of time
R	R	Two-body separation
R_0	R0	Initial value of the two-body separation
γ	GAMMA	Relative perturbation

Modification of COMMON

(a) Constant size

Existing dummies ..., *XDUM*(10), *NDUM*(10)

New variables *XNEW*(2), *NEW*

..., *XNEW*(2), *XDUM*(8), *NEW*, *NDUM*(9)

(b) Enlargement

Increase COMMON *COMMON/EXTRA/ A*(5), *B*, *NEW*(6)

Add to MYDUMP *REAL * 4 XNEW*

New COMMON *COMMON/EXTRA/ XNEW*(18)

Add READ/WRITE ..., *XNEW*

Correction Procedures

Escape removal	update arrays, energy & N
Single mass loss	potential energy correction
Binary mass loss	$ma = \text{const}$ & energy update
Collisions	energy & particle update
Supernovae	velocity kick
Differential force	c.m. approximation for chain
Tidal corrections	configuration change
New polynomials	re-initialization on mass loss
Total energy	constant of the motion
Correction terms	$E_{\text{kin}}, E_{\text{pot}}, E_{\text{tide}}, E_{\text{bin}}, E_{\text{sub}},$ $E_{\text{merge}}, E_{\text{coll}}, E_{\text{mdot}}, E_{\text{kick}}$

Getting Started

1. Download code `nbody6.tar.gz`
2. Unzip `gunzip nbody6.tar.gz`
3. Extract files `tar xvf nbody6.tar`
4. Check `params.h` `NMAX, LMAX, KMAX, MMAX`
5. Compile the code `make nbody6`
6. Create run directory `mkdir Run`
7. Run test input `time nbody6 <input >output &`
8. Profiling `Makefile with -O3 -pg`
9. Performance data `gprof nbody6 gmon.out -p >OUT`

NBODY6 Profiling

Membership	$N = 5000$	$N = 10000$	$N = 20000$
REGINT	90.0	93.6	95.8
NBINT	7.0	4.2	2.6
INTGRT	1.5	1.0	0.7
ENERGY	0.8	0.7	0.6
TSTEP	0.5	0.3	0.2

NBODY6 with GPU

CNBINT	43.4
INTGRT	31.0
NBINTP	17.0
GPUNB	2.8
TSTEP	2.4

Typical Input File

```
1 1000000.0
10000 1 20 79000 295 1
0.02 0.02 0.18 2.0 10.0 1000.0 3.0E-05 2.0 0.6
2 0 0 0 1 0 1 3 0 0
0 0 0 1 1 2 1 0 3 4
1 0 2 0 0 2 2 1 0 1
1 1 2 2 1 1 0 2 0 3
0 0 0 0 0 0 0 0 0 0
1.0E-05 0.0006 0.2 1.0 1.0E-06 0.01
2.3 12.0 0.2 500 0 0.02 0.0 100.0
0.5 0.0 0.0 0.0
0.0002 -1.0 1.0 10.0 0 0 0
0.125D0
```